Class Exercise 2 Solution

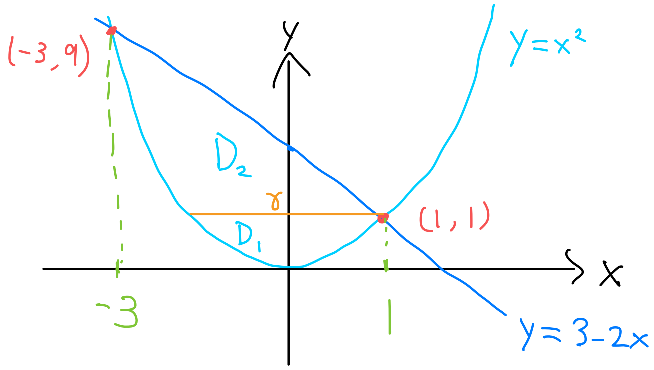
1. Reverse the order of integration in the iterated integral

$$\int_{-3}^{1}\int_{x^2}^{3-2x} f(x,y)\,dydx.$$

Sketch the figure first. The answer should be the sum of two integrals.

Solution. The curves $y = x^2$ and $y = 3 - 2x$ intersects at $(1,1)$ and $(-3,9)$. After reversing the order of integration, we have

$$\int_0^1\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)\,dxdy + \int_1^9\int_{-\sqrt{y}}^{\frac{1}{2}(3-y)} f(x,y)\,dxdy\ .$$

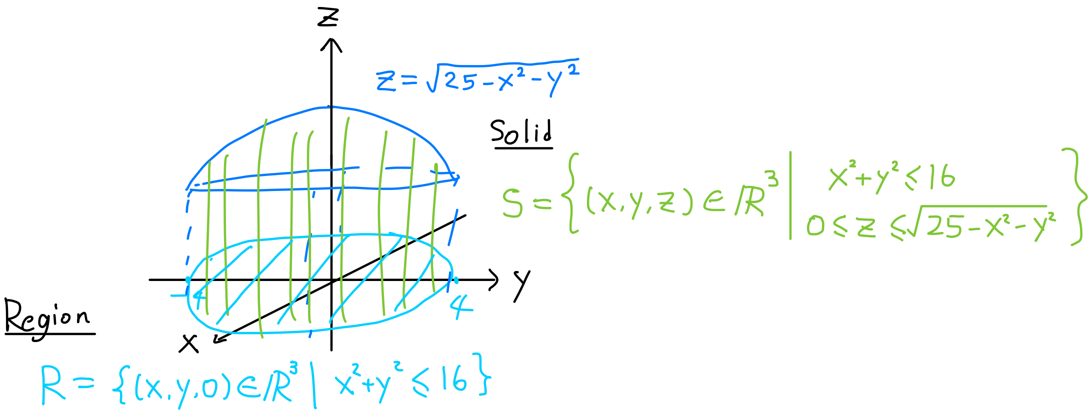


2. Sketch the region of integration and the solid whose volume is given by the double integral

$$\int_{-4}^{4}\int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \sqrt{25-x^2-y^2}\,dydx\ .$$

No need to evaluate the integral.

Solution. The region of integration is the disk $x^2+y^2 \le 16$. This integral yields the volume of the set bounded by the graph of $z = \sqrt{25-x^2-y^2}$ and the xy-plane over the disk.



3. Find the area of the region bounded by the lines $y = 2x, y = x/2$ and $y = 3 - x$.

Solution. The lines $y = 2x, y = 3 - x$ intersect at $(1,2)$. The lines $y = x/2, y = 3 - x$ intersect at $(2,1)$. The area bounded by them is given by

$$\int_0^1\int_{x/2}^{2x} dydx + \int_1^2\int_{x/2}^{3-x} dydx\ = \frac{3}{4}+\frac{3}{4} = \frac{3}{2}\ .$$